CORRECTIONS TO THE IMPULSE APPROXIMATION

Khin Maung Maung
Department of Physics
Old Dominion University
Norfolk, VA 23508-8507, USA

and

L. W. Townsend
NASA Langley Research Center
Hampton, VA 23665-5225, USA

and

P. A. Deutchman
Department of Physics
University of Idaho
Moscow, ID 83843, USA

Abstract: Corrections to the first order optical potential with the impulse approximation are presented. It is demonstrated that the corrections resulting from replacement of the pseudo, two-body transition operators appearing in either the Watson or the Kerman-McManus-Thaler formalisms are comparable in size to the multiple scattering corrections. Numerical evaluation of the on-shell matrix elements of these correction terms are performed and energy dependencies studied.

1. Introduction

Since the appearance of Watson's multiple scattering paper[1] there have been numerous formalisms and calculational developments in the field of hadron-nucleus scattering. Among these, the Kerman-McManus-Thaler (KMT) formalism[2] has been the most widely used. Because the multiple scattering series for an optical potential has an infinite number of terms, described through the use of suitably chosen pseudo two-body operators, it must be truncated for practical calculations. The most frequently used approximation, which keeps only the first term of the multiple scattering series, is known as the single scattering approximation. The double scattering and higher order terms are then neglected on the premise that they correspond to two-particle correlations and higher, which are usually assumed to be small. A further approximation is often made in which the pseudo two-body operator is replaced by the free two-body t-matrix. This approximation, known as the impulse approximation, is believed to be valid for high incident energies. However, if we wish to perform an optical potential calculation beyond the impulse approximation, the two lowest order corrections that should be considered are: (i) the correction due to the replacement of the pseudo two-body operator by the free t-matrix, namely the propagator correction or impulse correction, and (ii) the next highest order term in the multiple scattering series, namely the double scattering term. In the literature, there is a tendency to include either but not both corrections [3,4].

In this paper, we show that these two corrections are comparable in size for either the Watson or the KMT formalism; therefore, we should include both of them in any calculation beyond the first-order impulse approximation. This point was first discussed by Saperstein [5] for the Watson formalism. We derive expressions for these corrections in both the Watson and KMT

formalisms. We also demonstrate that, under the extreme closure approximation (neglect of the target nucleon momentum variable in the intermediate states), the single scattering, or the first-order optical potential with the impulse approximation, has a smaller total correction in the KMT formalism than in the Watson formalism. At this point, we note that Tandy et al. [4], using a three-body model for the first-order optical potential, obtained results indicating that the first order Watson optical potential could be closer than KMT to the impulse approximation result; however, they neglected consideration of double scattering. In this work, we are interested only in the relative sizes of the correction terms compared to the first-order impulse approximation. Hence, we will make no direct comparison with data, since obtaining a close fit to the data is highly model dependent in the sense that it depends upon the choice of t-matrix and the target density function. Instead, we choose to study the correction terms in a manner that is less model dependent. For this work, we assume the closure and factorization approximations to be valid. Nonrelativistic kinematics are used.

2. Multiple Scattering Series For The Optical Potential

In this section, we briefly rederive, for completeness, the multiple scattering series of the optical potential in both the Watson and KMT formalisms. We will first derive the Watson optical potential by starting with the (A+1)-body Lippmann-Schwinger equation for the transition operator T,

$$T = V + V\underset{\sim}{a}\, G_0 T \ , \tag{1}$$

where the (A+1)-body unperturbed Green function G_0 is given by

$$G_o = (E - h_o - H_A + i\eta)^{-1} \quad . \tag{2}$$

Here h_o and H_A are the projectile kinetic energy operator and A-body target Hamiltonian, respectively. V is the residual interaction between the projectile and the target nucleons, e.g., $V = \sum_{i=1}^{A} v_{oi}$, where v_{oi} is the two-body interaction between projectile and the i^{th} target particle and A is the number of target particles. In (1) we have displayed the target antisymmetrization operator $\underset{\sim}{a}$ explicitly. It is a reminder that when expanding the many-body Green's function G_o in the intermediate states, we should only use the properly antisymmetrized target states. To obtain the elastic scattering equation, we define the projectors P and Q by

$$P = |\Phi_o\rangle \langle\Phi_o| \quad , \tag{3}$$

and

$$\underset{\sim}{a} = P + Q \quad , \tag{4}$$

where Φ_o is the antisymmetrized target ground state. Using (3) and (4), we can rewrite (1) in the form of two coupled equations as

$$T = U + U\, P\, G_o\, P\, T \quad , \tag{5}$$

and

$$U = V + V\, Q\, G_o\, Q\, U \quad . \tag{6}$$

The operator U appearing in these last two equations is the optical potential operator, and (6) gives U in terms of the residual interaction V. To express (6) in a multiple scattering series form, we write U as $U = \sum_{i=1}^{A} U_{oi}$ so that (6) becomes

$$\sum_{i=1}^{A} U_{oi} = \sum_{i=1}^{A} v_{oi} + \sum_{i=1}^{A} v_{oi}\, Q\, G_o\, Q \sum_{j=1}^{A} U_{oj} \,. \tag{7}$$

Now define a pseudo two-body operator τ by

$$\tau_{oi} = v_{oi} + v_{oi}\, Q\, G_o\, Q\, \tau_{oi} \,, \tag{8}$$

and

$$\tau_{oi} = v_{oi} + \tau_{oi}\, Q\, G_o\, Q\, v_{oi} \,. \tag{9}$$

In terms of this operator τ, we can rewrite (7) as

$$U = \sum_{i=1}^{A} \tau_{oi} + \sum_{i=1}^{A} \tau_{oi}\, Q\, G_o\, Q \sum_{j\neq i}^{A} \tau_{oj} + \text{---} \,. \tag{10}$$

The last equation (10) is the multiple scattering series for the Watson optical potential which is to be used in (5) for the elastic scattering amplitude. To arrive at the comparable KMT result, we start with (1) by inserting $T = \sum_{i=1}^{A} T_{oi}$ to give

$$\sum_{i=1}^{A} T_{oi} = \sum_{i=1}^{A} v_{oi} + \sum_{i=1}^{A} v_{oi}\, \underset{\sim}{a}\, G_o \sum_{j=1}^{A} T_{oj} \,. \tag{11}$$

We now define the KMT pseudo two-body operator τ' by

$$\tau'_{oi} = v_{oi} + v_{oi} \underset{\sim}{a} G_o \tau'_{oi} \tag{12}$$

$$= v_{oi} + \tau'_{oi} \underset{\sim}{a} G_o v_{oi} \ .$$

In terms of τ'_{oi}, (11) then becomes

$$T = \sum_i \tau'_{oi} + \sum_i \tau'_{oi} \underset{\sim}{a} G_o \sum_{j\neq i} T_{oj} \ . \tag{13}$$

Since the matrix element of T must be taken between antisymmetric target states and the target intermediate states to be inserted are also antisymmetric, the matrix elements involved in (13) are symmetric. Therefore, the matrix elements of each τ_{oi}, i=1, 2, --, A are identical, and the $i\neq j$ restriction in (13) can be accounted for by a mere counting factor; hence, we arrive at

$$T = A\tau' + (A-1)\, \tau' \underset{\sim}{a} G_o T \ , \tag{14}$$

or

$$T' = (A-1)\tau' + (A-1)\, \tau' \underset{\sim}{a} G_o T' , \tag{15}$$

where T and T' are related by

$$T' = \frac{A-1}{A} T \ . \tag{16}$$

Inserting projectors P and Q defined in (3) and (4) gives

$$T' = U' + U' P G_o P T', \tag{17}$$

and

$$U' = (A-1)\,\tau' + (A-1)\,\tau' Q G_o Q U', \tag{18}$$

or

$$U' = \frac{A-1}{A} \sum_i \tau_{oi}' + \left(\frac{A-1}{A}\right)^2 \sum_i \tau_{oi}' Q G_o Q \sum_j \tau_{oj}' + \cdots . \tag{19}$$

Equation (19) is the multiple scattering series for the KMT optical potential to be used in (17). The physical scattering amplitude is obtained by rescaling with the (A/A-1) factor.

Although these two formalisms are different in appearance (e.g., the pseudo two-body operators τ and τ' and the counting factor coefficients), the physics contained in them is the same. Thus, if we solve (5) with (10) as input, we expect the same result as would be obtained by solving (17) with (19) as input and multiplying the result by a factor (A/A-1). The relationships between these two formalisms are studied in ref. 6). Although these two formalisms are expected to give the same result, one must use the full optical potentials given by (10) and (19) in their respective scattering equations. In practice, it is impossible to use the full optical potential in either formalism. Therefore, the most common practice is to retain only the first term in the optical potential series which is usually referred to as the single scattering or the first-order optical potential (FOP). For the Watson case, the FOP is given by

$$U^{(1)} = \sum_{i=1}^{A} \tau_{oi} \ , \tag{20}$$

and for the KMT case by

$$U'^{(1)} = \frac{A-1}{A} \sum_{i=1}^{A} \tau'_{oi} \ , \tag{21}$$

where the two-body operators τ and τ' are given by (8) and (12), respectively. Clearly, it is then expected that, when truncating the optical potential to any finite order, the results of the two different formalisms will be different. Another common practice is the replacement of the pseudo two-body operator τ or τ' by the free two-body t-matrix, which is known as the impulse approximation. In this approximation, the Watson FOP becomes

$$U_{Im}^{(1)} = \sum_{i=1}^{A} t_{oi} \ , \tag{22}$$

and for KMT it is

$$U_{Im}'^{(1)} = \frac{A-1}{A} \sum_{i=1}^{A} t_{oi} \ , \tag{23}$$

where the free two-body t-matrix is defined as

$$\begin{aligned} t_{oi} &= v_{oi} + v_{oi} \frac{1}{e-h_o-h_1+i\eta} t_{oi} \\ &= v_{oi} + t_{oi} \frac{1}{e-h_o-h_1+i\eta} v_{oi} \ . \end{aligned} \tag{24}$$

It is interesting to note that in this approximation, i.e., FOP under the impulse approximation, U and U' differ only by a scaling factor. Since U and U' given by (22) and (23) are the most commonly used, it is useful to investigate how accurate these approximations are. We know, in general, that FOP with the impulse approximation gives reasonably accurate predictions at relatively high energies ($\sim$ 400 MeV). Does this occur, however, because the multiple scattering corrections are negligible and the differences between free t and τ or τ' are small, or because the total correction itself is small? It would also be interesting to know at what energies either correction is most important. The best way is of course to compare the FOP U and U' given by (22) and (23) with their respective full optical potentials. Since it is impossible to use the full optical potential we will assume that the multiple scattering series for U and U' given in (10) and (19) converge rapidly enough that the first two terms give us a valid approximation to the full optical potential. This is reasonable because the third and higher order terms correspond to quantities involving three and more particle correlation functions, which are expected to be much smaller.

In terms of the free t-matrix, U and U', up to the second order in t, can be written as

$$U \simeq At + At\,(QG_0Q - g_0)t + A(A-1)tQG_0Qt\ , \tag{25}$$

and

$$U' \simeq (A-1)t + (A-1)t(G_0 - g_0)t + (A-1)^2 tQG_0Qt\ . \tag{26}$$

Here g_0 is the free two-body propagator appearing in (24). In (25) and (26), the respective second terms on the right sides correspond to the propagator or impulse correction arising from the use of free t, and the third terms correspond to the double scattering correction. We will demonstrate in this work that these two types of corrections are not negligible themselves but, they are comparable in size and there are some effective cancellations between them. Corrections due to the antisymmetrization between projectile and target nucleons [7,8] are not considered since the nature and origin of this correction is different. It should also be noted that we are working in the multiple-scattering formalism in which we do not include the projectile-target antisymmetry from the beginning. Our only purpose is to study the consequences of using the FOP under the impulse approximation.

3. Evaluation Of The Potentials

In order to evaluate the optical potentials given by (25) and (26), we form matrix elements of U and U' between initial and final elastic channel states; $\langle U'' \rangle \equiv \langle \underset{\sim}{K}' \Phi_0 | U'' | \underset{\sim}{K} \Phi_0 \rangle$, where U'' is either U or U'. We then evaluate four different types of matrix elements, viz. $\langle t \rangle$, $\langle t g_0 t \rangle$, $\langle t G_0 t \rangle$ and $\langle tQ\, G_0 Q t \rangle$. In evaluating these matrix elements we use a local t-matrix and we also employ the extreme closure approximation [2], in which we neglect the struck nucleon energy and the excited state target energies in the energy denominator. As a result, we can use a form factor sum rule [2]

$$\sum_n \rho_{on}(\underset{\sim}{q})\, \rho_{no}(\underset{\sim}{q}') = \frac{1}{A}\rho_{oo}(\underset{\sim}{q}+\underset{\sim}{q}') + \frac{A-1}{A}\left[C_{oo}(\underset{\sim}{q},\underset{\sim}{q}') + \rho_{oo}(\underset{\sim}{q})\,\rho_{oo}(\underset{\sim}{q}')\right] \quad , \quad (27)$$

where $\rho(\underset{\sim}{q})$ is the Fourier transform of the single particle density $\rho(\underset{\sim}{r})$, and $C(\underset{\sim}{q}, \underset{\sim}{q}')$ is the Fourier transform of the pair correlation function $C(\underset{\sim}{r}, \underset{\sim}{r}')$ given by

$$C_{oo}(\underset{\sim}{r}, \underset{\sim}{r}') = \rho_{oo}(\underset{\sim}{r}, \underset{\sim}{r}') - \rho_{oo}(\underset{\sim}{r})\, \rho_{oo}(\underset{\sim}{r}') \quad . \tag{28}$$

Here $\rho_o(\underset{\sim}{r}, \underset{\sim}{r}')$ is the two-particle density. After employing a local t-matrix, extreme closure approximation, and the sum rule given in (27), we arrive at

$$\langle t \rangle = t(\underset{\sim}{q})\, \rho_{oo}(\underset{\sim}{q}) \ , \tag{29}$$

$$\langle t g_o t \rangle = I_1(\underset{\sim}{q})\, \rho_{oo}(\underset{\sim}{q}) \ , \tag{30}$$

$$\langle t G_o t \rangle = \frac{1}{A}\, \rho_{oo}(\underset{\sim}{q})\ I_1(\underset{\sim}{q}) + \frac{A-1}{A}\, [I_2(\underset{\sim}{q}) + I_3(\underset{\sim}{q})] \ , \tag{31}$$

and

$$\langle t Q G_o Q t \rangle = \frac{1}{A}\, \rho_{oo}(\underset{\sim}{q}) I_1(\underset{\sim}{q}) + \frac{A-1}{A}\, I_3(\underset{\sim}{q}) - \frac{1}{A}\, I_2(\underset{\sim}{q}) \quad . \tag{32}$$

The functions $I_1(\underset{\sim}{q})$, $I_2(\underset{\sim}{q})$ and $I_3(\underset{\sim}{q})$ are defined as

$$I_1(\underset{\sim}{q}) = \int \frac{t(\underset{\sim}{k}', \underset{\sim}{k}'')\, t(\underset{\sim}{k}'', \underset{\sim}{k})}{E_k - E_{k''} + i\eta}\, d\underset{\sim}{k}'' \ , \tag{33}$$

$$I_2(\underset{\sim}{q}) = \int \frac{t(\underset{\sim}{k}', \underset{\sim}{k}'')\, \rho_{oo}(|\underset{\sim}{k}' - \underset{\sim}{k}''|)\ \ t(\underset{\sim}{k}'', \underset{\sim}{k})\, \rho_{oo}(|\underset{\sim}{k}'' - \underset{\sim}{k}|)}{E_k - E_{k''} + i\eta}\, d\underset{\sim}{k}'' \ , \tag{34}$$

and

$$I_3(\underset{\sim}{q}) = \int \frac{t(\underset{\sim}{k}', \underset{\sim}{k}'')\ t(\underset{\sim}{k}'', \underset{\sim}{k})\ C_{oo}(\underset{\sim}{k}' - \underset{\sim}{k}'', \underset{\sim}{k}'' - \underset{\sim}{k})}{E_k - E_{k''} + i\eta} , \tag{35}$$

where the momentum transfer $\underset{\sim}{q} = \underset{\sim}{k}' - \underset{\sim}{k}$. It should be noted that in (32) we have kept the terms proportional to the $\frac{1}{A}$ factor. Because the coefficient of $\langle t\ Q\ G_o\ Q\ t \rangle$ is $(A-1)^2$ in the KMT case and $A(A-1)$ in the Watson case, these terms become comparable in size to the impulse correction terms. Now from (25) and (26) we have

$$\langle U \rangle = A\ t(\underset{\sim}{q})\ \rho_{oo}(\underset{\sim}{q}) + \{(A-1)\ [I_3(\underset{\sim}{q}) - I_1(\underset{\sim}{q})\ \rho_{oo}(\underset{\sim}{q})] - I_2(\underset{\sim}{q})\}$$

$$+ \{(A-1)\ [I_1(\underset{\sim}{q})\ \rho_{oo}(\underset{\sim}{q}) - I_2(\underset{\sim}{q}) + (A-1)\ I_3(\underset{\sim}{q})]\} , \tag{36}$$

and

$$\langle U' \rangle = (A-1)t(\underset{\sim}{q})\ \rho_{oo}(\underset{\sim}{q}) + \{\frac{(A-1)^2}{A}\ [I_2(\underset{\sim}{q}) + I_3(\underset{\sim}{q}) - I_1(\underset{\sim}{q})\ \rho_{oo}(\underset{\sim}{q})]\}$$

$$+ \{\frac{(A-1)^2}{A}\ [I_1(\underset{\sim}{q})\ \rho_{oo}(\underset{\sim}{q}) + (A-1)\ I_3(\underset{\sim}{q}) - I_2(\underset{\sim}{q})]\} . \tag{37}$$

In each of the last two equations, the terms in the first set of curly brackets corresponds to the propagator correction, and the terms in the second curly brackets correspond to the double scattering correction. We note here that these two corrections are of comparable size and there will be some cancellations between them in both the Watson and KMT optical potentials. We emphasize that these cancellations result from the use of the extreme closure

approximation. If the extreme closure approximation were not used, only partial cancellation could be expected. After the cancellations, we have

$$\langle U \rangle = At(\underset{\sim}{q})\ \rho_{oo}(\underset{\sim}{q}) + A\ [(A - 1)\ I_3(\underset{\sim}{q}) - I_2(\underset{\sim}{q})]\ , \tag{38}$$

and

$$\langle U' \rangle = (A - 1)t(\underset{\sim}{q})\ \rho_{oo}(\underset{\sim}{q}) + (A - 1)^2 I_3(\underset{\sim}{q})\ . \tag{39}$$

It is interesting to note that the total correction for the first-order KMT depends only on the function $I_3(\underset{\sim}{q})$ which involves the two particle correlation which, for an uncorrelated nucleus, vanishes. Therefore it could be a possible explanation for the success of first-order KMT with impulse approximation for medium and high energy regions.

4. Results And Discussion

We now compare the first-order approximation to the more complete forms given by (38) and (39) which include the propagator and double scattering corrections. Of course one should consider the off-shell and non-local effects in order to make a complete comparison. Since we have chosen a local t, the expressions for $\langle U \rangle$ and $\langle U' \rangle$ in (38) and (39) are already local. Such local representations of the correction terms may not be a valid approximation for low energies, but nevertheless, for this initial study, we assume them to be adequate. Because the on-shell and near on-shell matrix elements of the optical potential give the major contribution to the scattering amplitude, we will concentrate our study on the on-shell matrix elements. In what follows, we calculate the terms involved in (38) and (39)

by employing a spin-isospin averaged local t-matrix[9]) and a Gaussian shape nuclear form factor. Since we do not intend to make a detailed comparison with experimental data, but are interested only in the overall effects of the correction terms, we feel justified in the use of this simple t-matrix and nuclear form factor. The free N-N t-matrix used is of the form[9])

$$t(q) = C(e,\sigma,\alpha)e^{-Bq^2} , \tag{40}$$

where $C(e,\sigma,\alpha)$ is a complex quantity depending on e, the N-N C-M energy; σ, the total N-N cross section; and α, the real-to-imaginary-part ratio of the forward scattering amplitude. For the nuclear form factor, we use the Fourier transform of a harmonic-well shape density[9]) given by

$$\rho(r) = \rho_0 \left[1 + \gamma \left(\frac{r}{a}\right)^2\right] e^{-r^2/a^2} . \tag{41}$$

The correlation function in configuration space is chosen to be [10, 11]

$$C(\bar{r}, \bar{r}') = \rho(\bar{r})\, \rho(\bar{r}')\, G(|\bar{r} - \bar{r}'|) , \tag{42}$$

with the function $G(|\bar{r}' - \bar{r}|)$ given in a parameterized form. Detailed expressions of these functional forms and parameters can be found in refs. 9, 10, and 11.

In figs. 1 and 2, we show the on-shell optical potential as a function of momentum transfer $\bar{q}$. In fig. 1 the curve denoted by UW1 corresponds to the first-order approximation for the Watson optical potential, UW12 corresponds to the potential with impulse correction only, UW13 refers to the one with double scattering correction alone, and UW123 contains both types of corrections and corresponds to the expression given by (38). Similar

notation is used for the KMT potential in fig. 2. In both figs. 1 and 2, the on-shell matrix elements are calculated for a 100 MeV proton projectile incident on a ^{16}O target nucleus. Serious overestimations or underestimations of the real and imaginary parts of the on-shell optical potentials can be seen for both Watson and KMT formalisms at this energy. It is obvious also that these corrections are of comparable magnitude, and they should both be included in any serious calculation beyond the use of the first-order impulse approximation. We also note here that the corrections become larger at higher momentum transfers.

In figs. 3, 4, 5, and 6, we show the percentage difference between the on-shell optical potential with both corrections denoted by $U^{(2)}$ and the one given by the first-order approximation denoted by $U^{(1)}$. The percentage difference is defined as

$$\Delta = \frac{U^{(2)} - U^{(1)}}{U^{(2)}} \times 100\% \ . \tag{43}$$

We show calculated results for 100, 200, and 300 MeV incident lab energies for the proton - ^{16}O optical potential. At 100 MeV the real part of the Watson potential gives a 16% difference at the forward angle and about 50% difference at $q = 2$ fm^{-1}. Although KMT results yield a smaller percentage difference in this momentum transfer range, it is obviously not negligible. This leads us to the conclusion that the first order (t_ρ) approximation is not valid at this energy for either formalism. As the incident energy increases, the percentage differences becomes smaller for both real and imaginary parts of the on-shell potential for each formalism. At 300 MeV, the real part of the KMT potential has less than a 1.7% difference in the range $q = 0$ to $q = 2$ fm^{-1}, while the Watson potential still gives an 8% to 16%

difference in the same momentum transfer range. Although this does not imply that the KMT formalism with the $t\rho$ approximation is absolutely reliable at this energy because of possible contribution from non-local and off-shell effects, it suggests that the $t\rho$ (KMT) approximation is reasonable at this energy. Note also that, although the percentage difference in Watson on-shell potential decreases as the incident energy increases, there remains non-negligible contributions from the correction terms even at 300 MeV. We also note here that the KMT formalism always gives a smaller total correction at all energies. It must be emphasized that all our conclusions rely upon the extreme closure approximation assumption and the fact that off-shell and non-local effects were not considered. We suggest that a full scattering calculation should be performed which includes both impulse and double scattering corrections simultaneously and consistently. Work on this has been initiated.

5. Conclusion

It has been shown that, in the multiple scattering formalisms for the optical potential, the propagator correction to the impulse approximation and the double scattering correction are of the same order. If the extreme closure approximation is assumed, then sizable cancellations between these two types of corrections occur. Therefore, it may be incorrect to include only one type of correction in any calculation beyond first-order under the impulse approximation. We calculated the on-shell optical potential for both the Watson and the KMT formalisms and demonstrated that for the extreme closure approximation, the KMT potential has a smaller correction. A complete scattering calculation incorporating the above-mentioned corrections is in progress.

6. Acknowledgements

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Figure Captions

Fig. (1) - The real part of the Watson optical potential with different correction terms. Notation is explained in the text.

Fig. (2) - The real part of the KMT optical potential with different correction terms. Notation is explained in the text.

Fig. (3) - The percentage differences Δ as defined by Eq. (43) vs $\bar{q}$ the momentum transfer. Δ for the Watson and KMT potentials are shown for the case of 100 MeV nucleon on ^{16}O.

Fig. (4) - Same as Fig. (3), but for 200 MeV.

Fig. (5) - Same as Fig. (3), but for 300 MeV.

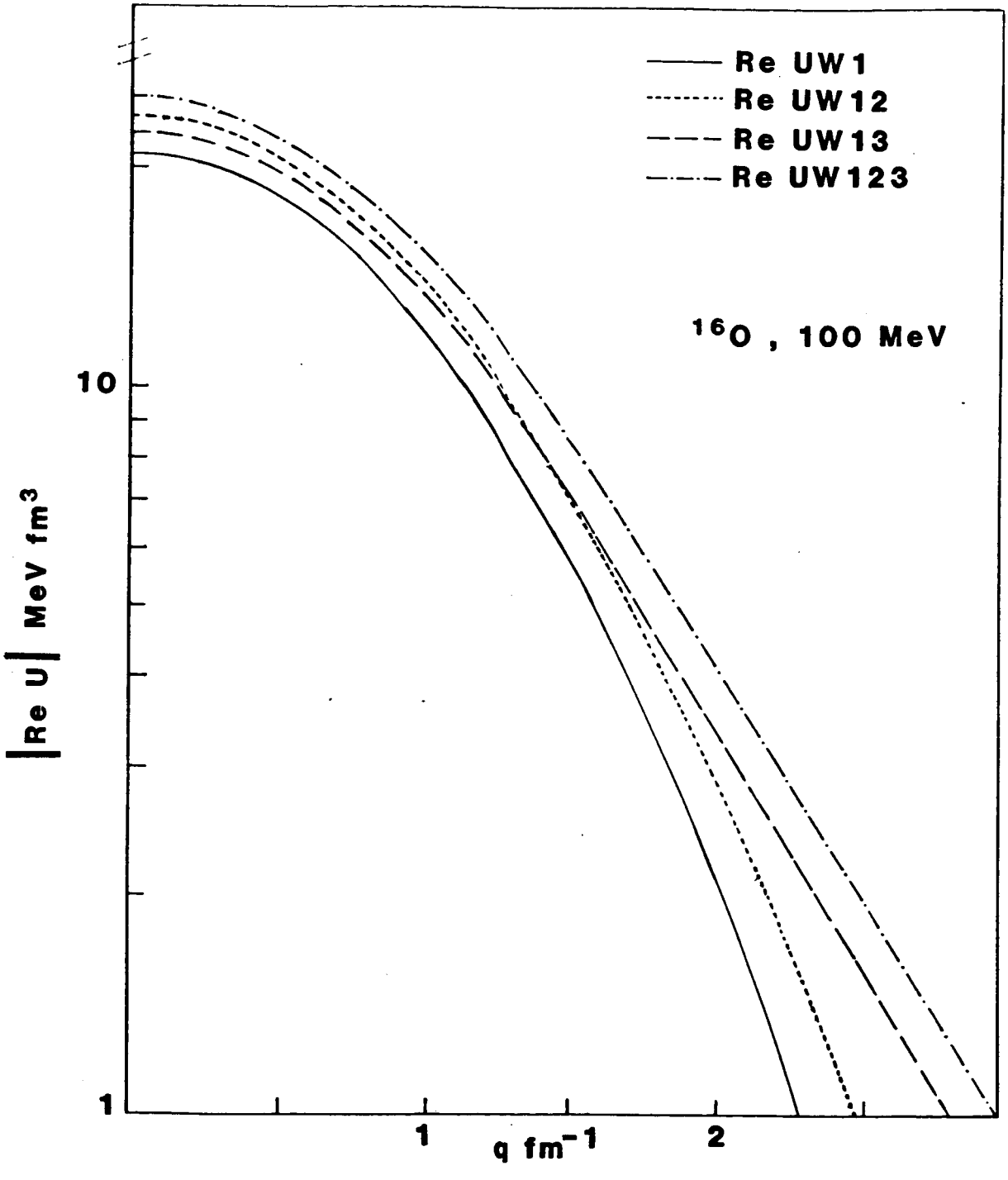
Re UW1
Re UW12
Re UW13
Re UW123
^{16}O , 100 MeV
10
1
|Re U| MeV fm^3
1
2
q fm^{-1}

Fig (1)

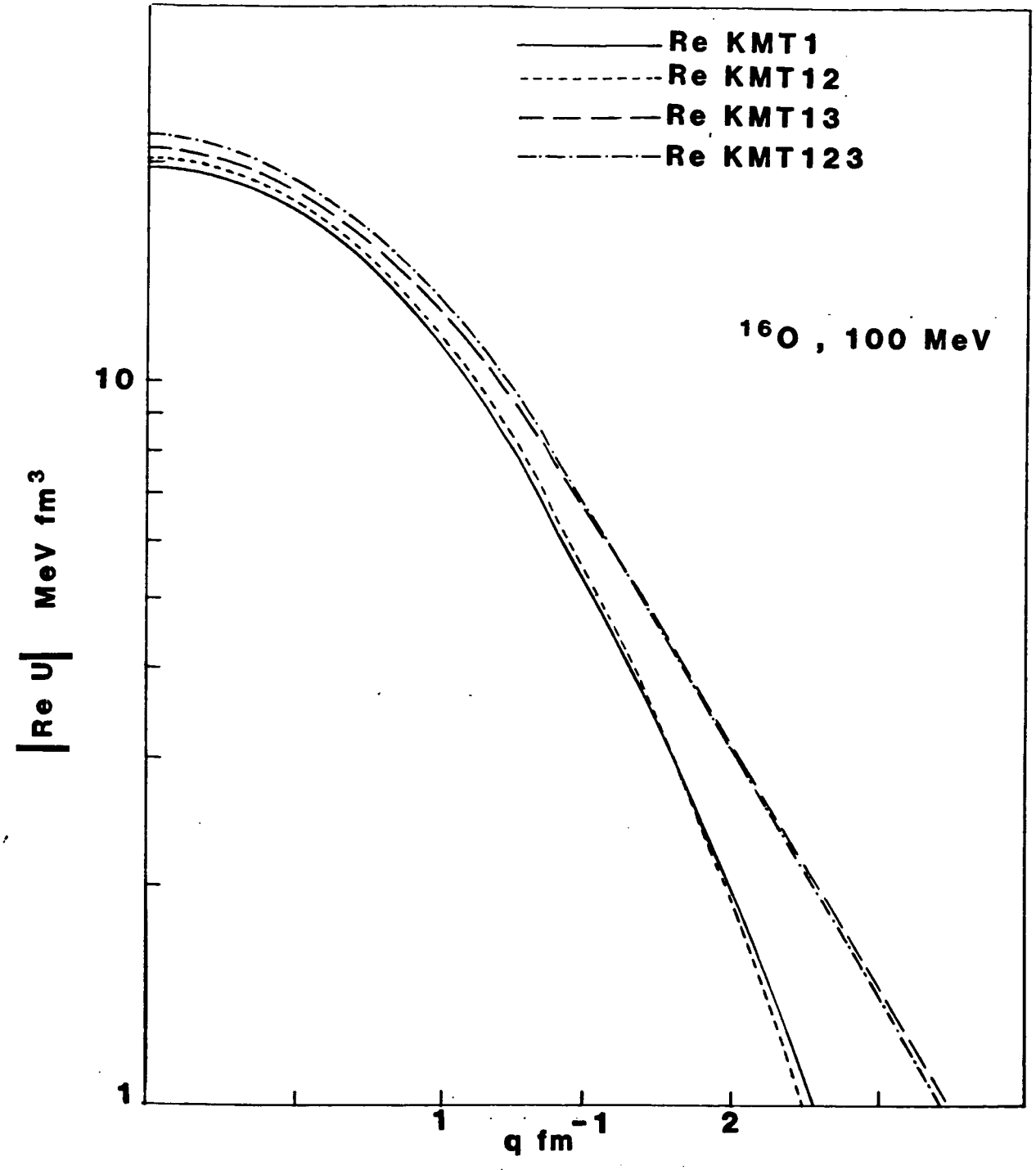
Re KMT1
Re KMT12
Re KMT13
Re KMT123
^{16}O , 100 MeV
10
1
|Re U| MeV fm^3
1
2
q fm^{-1}

Fig (2)

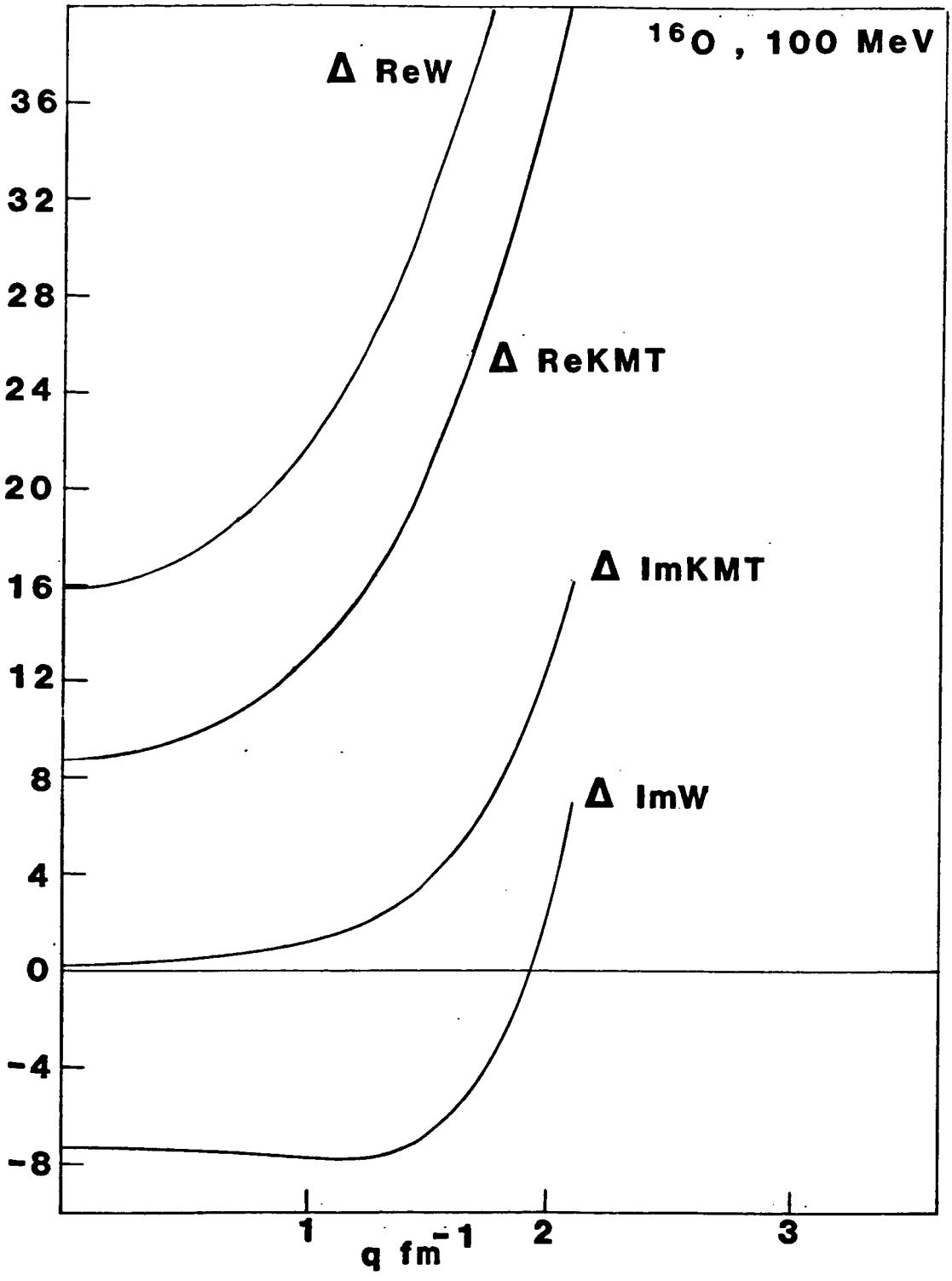
^{16}O , 100 MeV
Δ ReW
Δ ReKMT
Δ ImKMT
Δ ImW
36
32
28
24
20
16
12
8
4
0
-4
-8
1
2
3
q fm^{-1}

Fig (3)

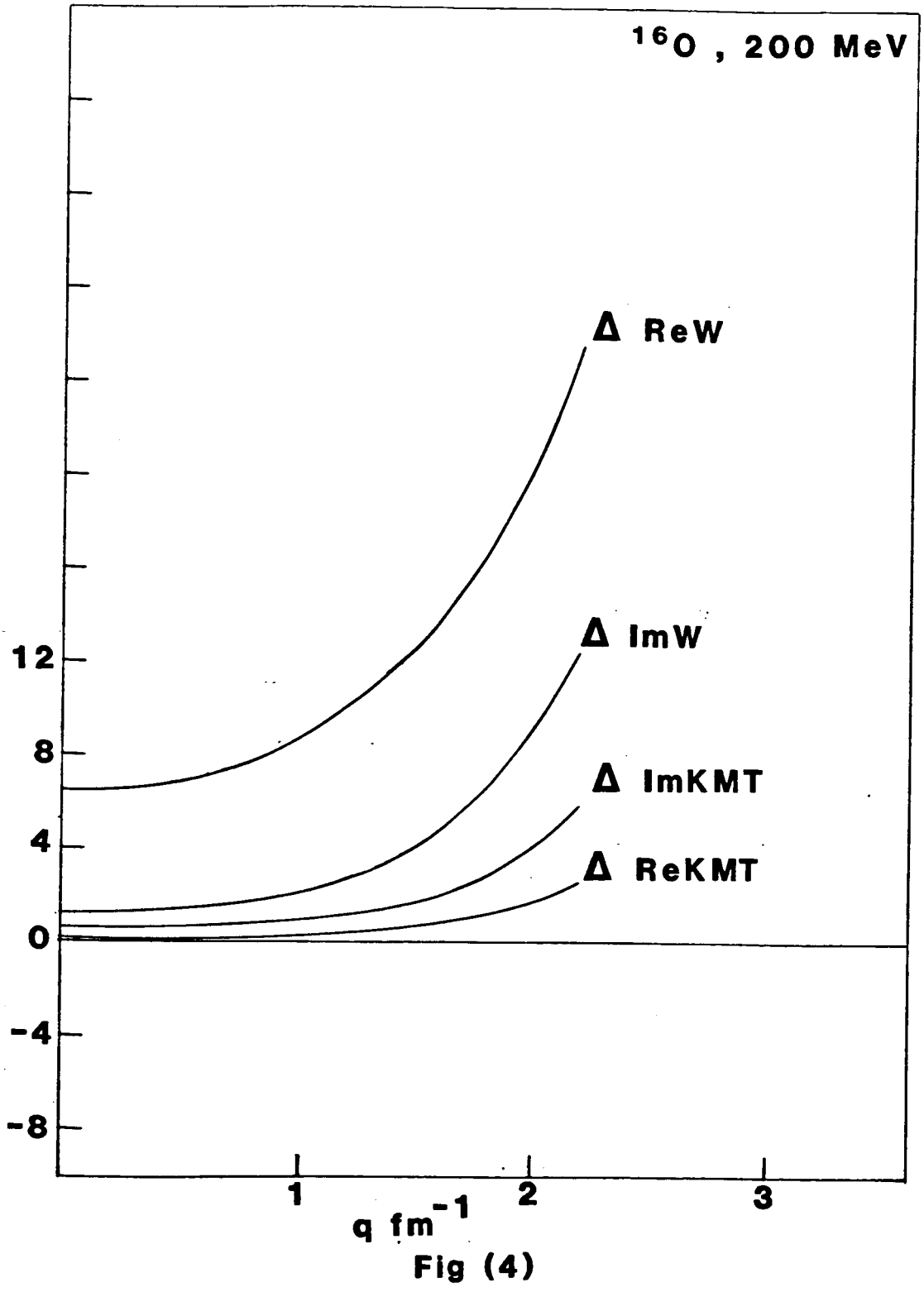

Fig (4)

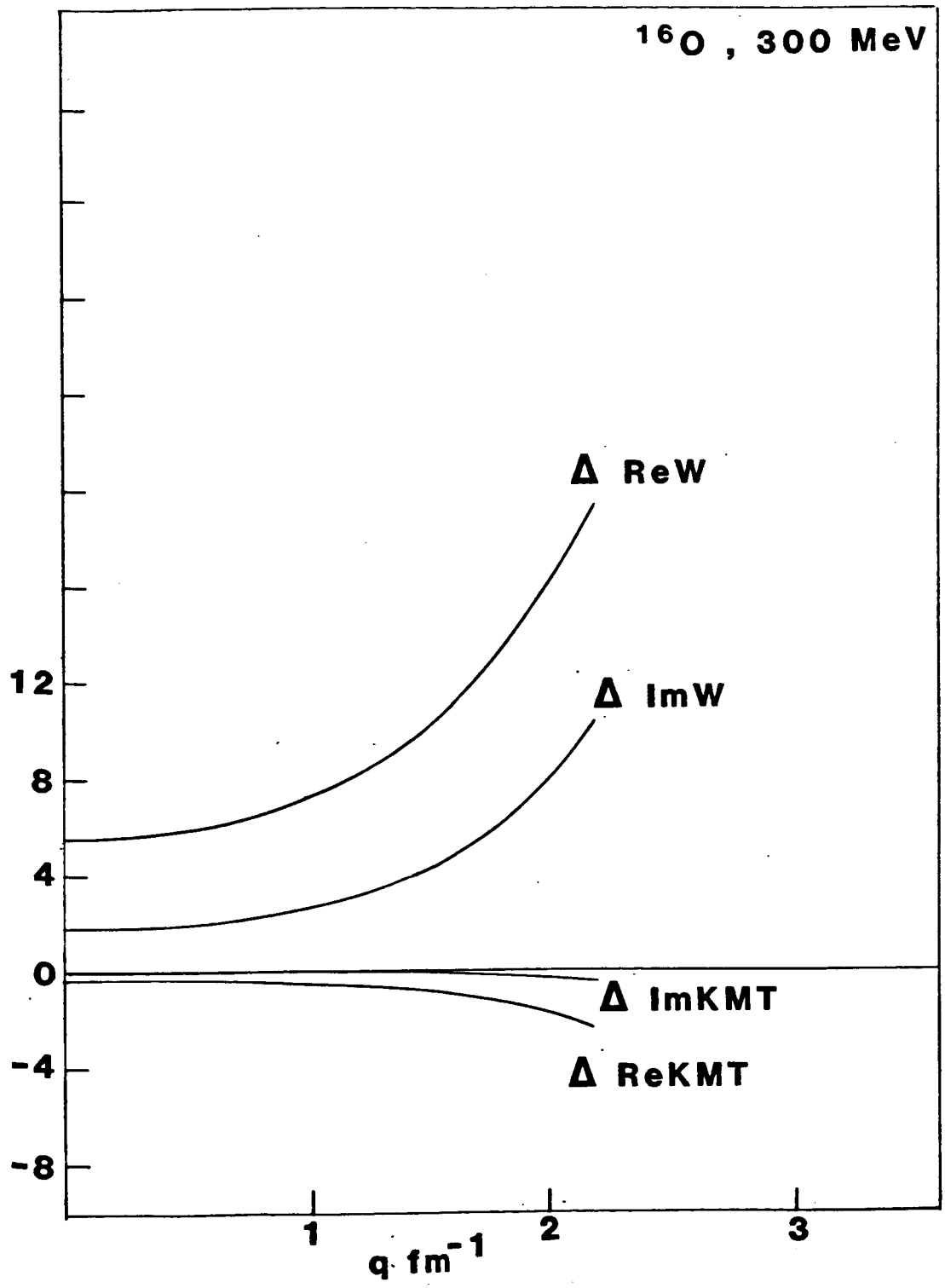

Fig (5)